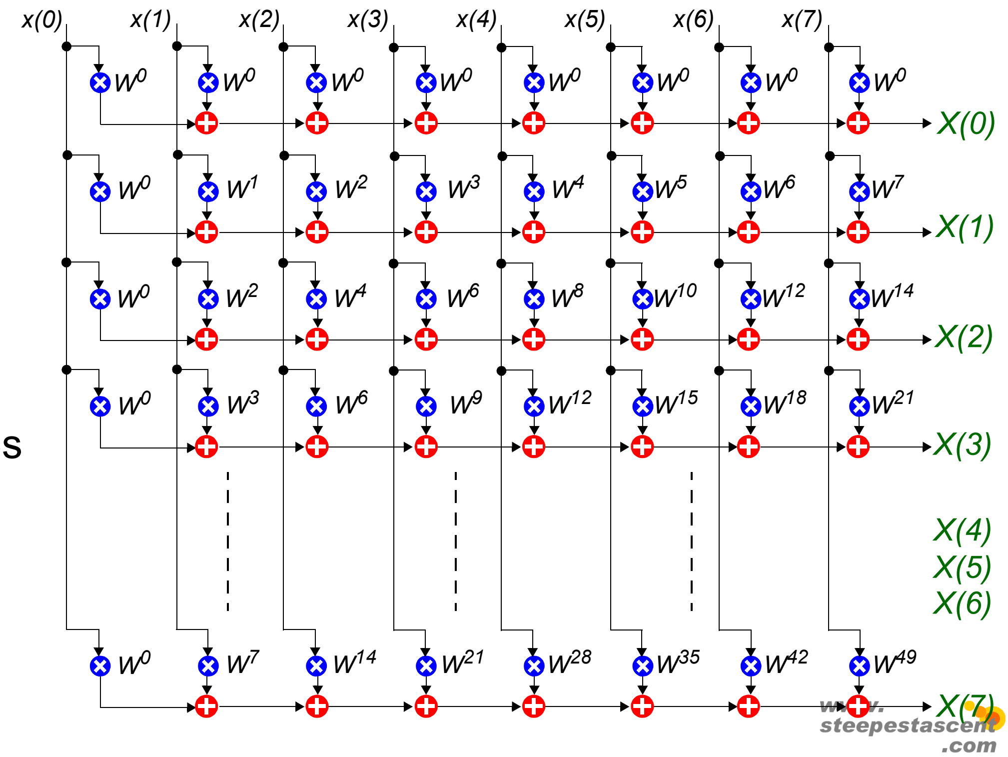
DFT Signal Flow Graph

$$X(k) = \sum_{n=0}^{N-1} x(n) W^{nk} \quad \text{where } W = e^{\frac{-j2\pi}{N}} \quad \textit{(to simplify figure)}$$

- $N^2 = 64$ Complex MACs



Notes:

Using the DFT algorithm to calculate the first four components of the DFT of a (trivial) signal with only 8 samples requires the following computations:

$$X(0) = x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7)$$
$$X(1) = x(0) + x(1)W^1 + x(2)W^2 + x(3)W^3 + x(4)W^4 + x(5)W^5 + x(6)W^6 + x(7)W^7$$
$$X(2) = x(0) + x(1)W^2 + x(2)W^4 + x(3)W^6 + x(4)W^8 + x(5)W^{10} + x(6)W^{12} + x(7)W^{14}$$
$$X(3) = x(0) + x(1)W^3 + x(2)W^6 + x(3)W^9 + x(4)W^{12} + x(5)W^{15} + x(6)W^{18} + x(7)W^{21}$$

However note that there is redundant (or repeated) arithmetic computation in the above equation. For example, consider the third term in the second line:

$$x(2)W^2 = x(2)e^{j2\pi\left(\frac{-2}{8}\right)} = x(2)e^{\frac{-j\pi}{2}}$$

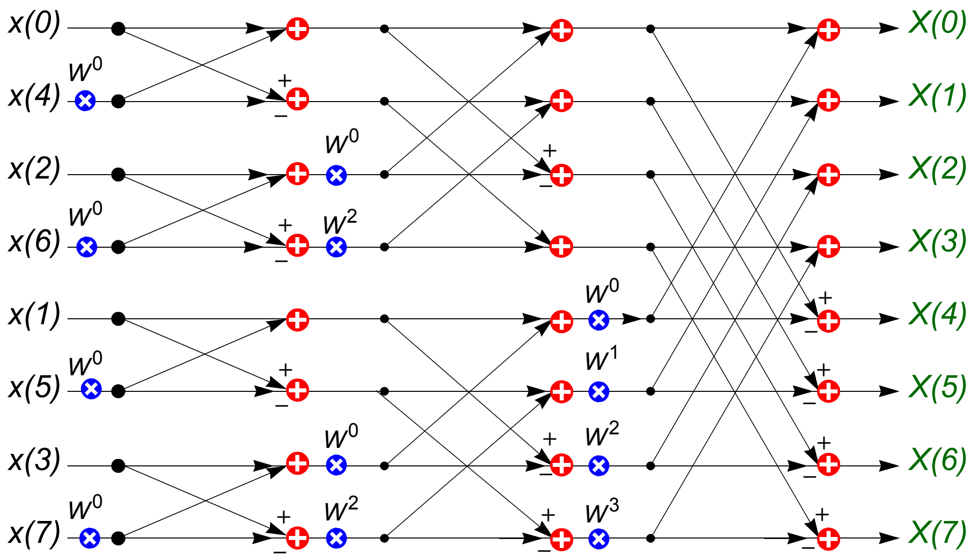
Now consider the computation of the third term in the fourth line:

$$x(2)W^6 = x(2)e^{j2\pi\left(\frac{-6}{8}\right)} = x(2)e^{\frac{-j3\pi}{2}} = x(2)e^{-j\pi}e^{\frac{-j\pi}{2}} = -x(2)e^{\frac{-j\pi}{2}}$$

Therefore we can save one multiply operation by noting that the term $x(2)W^6 = -x(2)W^2$. In fact because of the periodicity of W^{kn} every term in the fourth line is available from the computed terms in the second line of the equation. Hence a considerable saving in multiplicative computations can be achieved if the computational order of the DFT algorithm is carefully considered.

Fast Fourier Transform (FFT)

- Exploiting the periodicity of W, the DFT can be more efficiently computed using the FFT computation strategy:

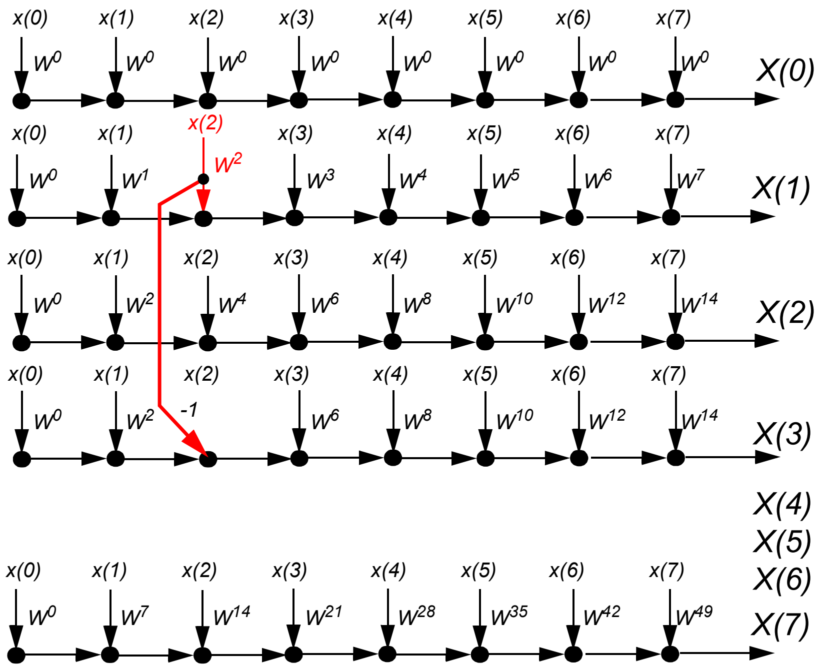


- $N\log_2 N = 24$
  Complex MACs

Decimation in Time - FFT

Notes:

The DFT signal flow graph, saving *one* multiplication.....



Modification of the signal flow graph to save the ***one*** multiplication highlighted in the previous Notes page. Repeating this process we can modify the DFT SFG into an FFT SFG (this graphical procedure is referred to as algorithmic engineering).